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Novel doorways and resonances in large-scale classical systems

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Abstract – We show how the concept of doorway states carries beyond its typical applications and usual concepts. The scale on which it may occur is increased to large classical wave systems. Specifically we analyze the seismic response of sedimentary basins covered by water-logged clays, a rather common situation for urban sites. A model is introduced in which the doorway state is a plane wave propagating in the interface between the sediments and the clay. This wave is produced by the coupling of a Rayleigh and an evanescent SP-wave. This in turn leads to a strong resonant response in the soft clays near the surface of the basin. Our model calculations are compared with measurements during Mexico City earthquakes, showing quite good agreement. This not only provides a transparent explanation of catastrophic resonant seismic response in certain basins but at the same time constitutes up to this date the largest-scale example of the doorway state mechanism in wave scattering. Furthermore the doorway state itself has interesting and rather unusual characteristics.

Introduction. – The concept of doorway state was introduced in nuclear physics a long time ago [1–4]. The doorway state mechanism is effective whenever a single or a few "distinct" states are coupled to the scattering channels as well as to a sea of more dense and complicated states, which themselves are much more weakly coupled to the continuum or not at all. Isobar analogue states are prime examples of doorway states; there the mechanism is based on weak symmetry breaking, combined with the fact that the symmetry is conserved or at least symmetry breaking is much weaker in the asymptotic channels. Beyond the nuclear problems, in which they were originally discovered, doorway states provide a concept that encompasses many physical systems.

Recently, doorway states have attracted renewed attention since they have been observed in atoms and molecules [5], clusters [6], quantum dots [7], and in C_{60} fullerenes [8,9]. In many scattering processes the doorway states provide the dominant mechanism to excite the more dense states from some incoming channel. Also one could find a relationship between the doorway state mechanism and the super-radiance effect, as is discussed in ref. [10]. Note that the doorway state is not an eigenstate of the whole system. The strength of the doorway state spreads among the eigenstates in some energy region and the strength function [3] describes the distribution of the excitation among the eigenstates. As a function of energy, the strength function follows, in the simplest case, a Breit-Wigner form, a Lorentzian, with a characteristic spreading width [3].

As mentioned above, doorway states can equally appear in classical wave systems, though now the frequency is involved rather than the energy. An indeed obvious example of a doorway state is given when there are physical spaces capable of resonating and separating the inner system from the continuum [11]. A membrane separating a resonator for sound waves from the open space is a useful toy model for the latter situation. Experimentally doorway states in classical waves were realized with flat microwave cavities. A rectangular cavity with a thin barrier inside was analyzed. This cavity presents a distinct state, which

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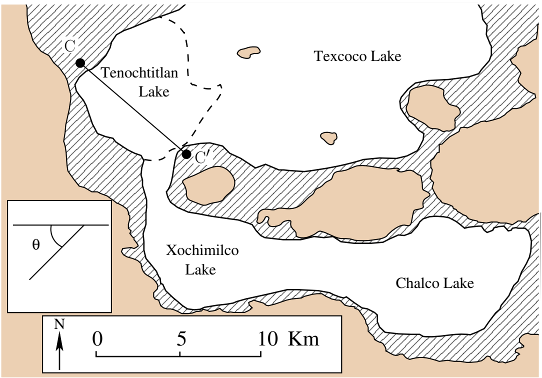

Fig. 1: (Color online) Schematic drawing of a geological structure that yields selective excitation through doorway states. In this alluvial basin the sediments (hatched regions) are surrounded by rock (colored zone) and covered by a narrow layer of soft clays (white regions). The S-wave enters into the system and produces a PR-wave formed by the coupling between a Rayleigh and an evanescent P-wave. Adapted from ref. [12].

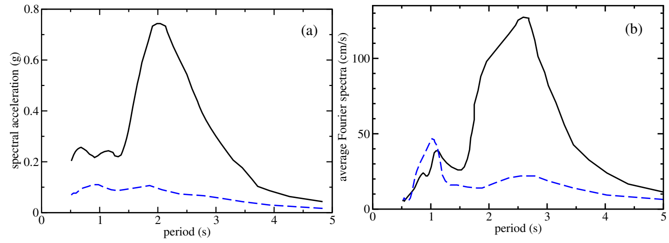

Fig. 2: (Color online) Spectral amplitude of the lake-bed clay in Mexico City referred to rock motion during two large earthquakes: (a) magnitude 8.1, 1985, September 19 and (b) magnitude 6.9, 1989, April 25. The continuous lines correspond to measurements in the lake zone while the dashed lines correspond to measurements in rock. Adapted from ref. [17].

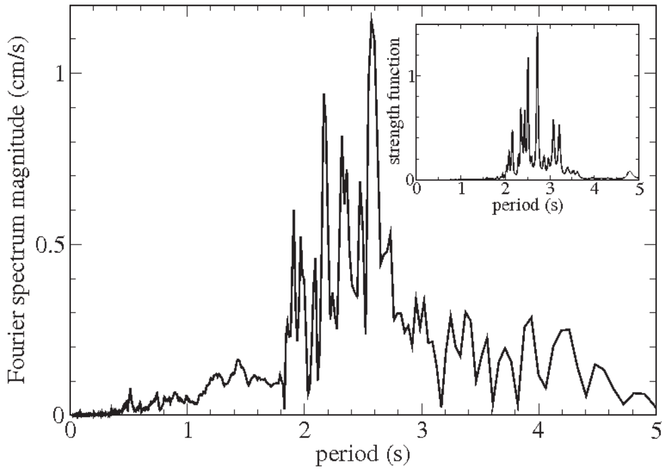

Fig. 3: Fourier spectrum magnitude of the lake-bed clay in Mexico City during the earthquake of magnitude 7.1, 1997, January 11 with improved resolution. In the inset the theoretical results of fig. 5 are shown.

was referred to as a superscar [13], and was found experimentally soon after [14]; it acts as the doorway state for the normal modes of the microwave cavity [15]. Due to this wide range of applicability one can say that doorway states provide a unifying concept in physics.

In this letter we show that doorway states are also relevant in natural classical systems; in particular, in the seismic response of sedimentary basins covered by soft clays. Note that the scale is four to five orders of magnitude larger than for the microwave experiments, and we are dealing with natural systems of great and very direct importance to humans. This geological structure is rather common and many cities are built on this type of basin, outstanding examples being Mexico City, San Francisco, Auckland, and Kyoto, among others. The similarities between these basins have long been emphasized [16]. The Mexico City basin, for example, consists of rather extended ($\sim 20\,\text{km}$) and deep ($\sim 1\,\text{km}$) alluvial deposits surrounded by steep mountain ranges on almost all sides. Within the alluvial sediment surface the basin contains water-logged and extremely soft clays partially covered by surface waters (remnants of the ancient lakes in the Valley of Mexico). The soft clays cover a region with reasonsably well-defined borders and are surrounded by harder sediments and rock outcrops (see fig. 1).

In what follows we will show that during earthquakes this geological structure leads to the existence of a distinct state strongly coupled to the incoming waves and to a sea of states surrounding it, while the direct coupling of the states in the soft-clay layer to incoming waves is so weak that it was neglected for many years. This "distinct state" is a Rayleigh wave formed at the interface between the harder deep sediments and the soft-clay layer. Such waves constitute the most effective coupling between incoming waves and local resonant states and, as we shall show, they meet the conditions of doorway states.

Earthquake measurements and model. – We begin by analyzing the observed spectral response during large earthquakes that impact such basins. This response is the equivalent of the strength function analyzed in the quantum-mechanical cases. The amplitude measured during two large earthquakes in Mexico City is shown in fig. 2 as a function of the period using low resolution [17]. They look like a textbook example of a resonant doorway state, see, for example, ref. [18], p. 206. Furthermore, when analyzed with a higher-frequency resolution the individual resonances become apparent. This can be seen in fig. 3 for the Fourier amplitude spectrum obtained in the large earthquake of 1997, January 11, in Mexico City with an array of accelerographs having improved resolution. It should be mentioned that similar Fourier spectra have also been observed in many different seisms.

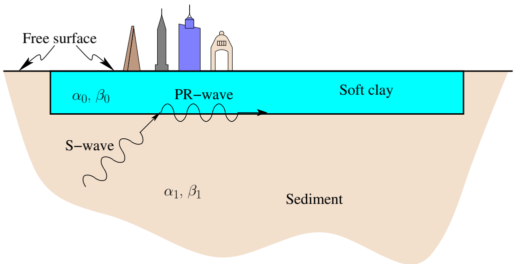

Fig. 4: (Color online) Schematic drawing of the geological structure that yields the strength function phenomenon. In alluvial basins the sediments are covered by a narrow layer of soft clays. The S-wave enters into the system and produces a PR-wave that results from the coupling of Rayleigh and evanescent P-waves.

On the other hand, a similar phenomenon has been observed in many instances in nuclear experiments [19].

In fig. 4 we depict schematically a cross-section of the Mexico City basin (indicated by the line CC′ in fig. 1) showing the sedimentary region on top of which lies a finite layer of soft clay. Mexico City, illustrated by typical buildings, is located on the free surface of the ancient Tenochtitlan lake. When an earthquake occurs, an elastic transverse wave (SV-wave) arrives at the sedimentary basin from below and generates a Rayleigh wave indicated as the PR-wave at the interface between the sediments and the soft clay [20]. In what follows α_1 indicates the velocity of the longitudinal elastic waves (P-waves) and β_1 the velocity of the S-waves in the sediment. Likewise, α_0 and β_0 indicate the velocities of P and S waves in the soft clay, respectively.

What is the doorway state in the seismic case? It has been known for a long time to geophysicists [21] that when a sedimentary layer is covered by a much softer material (such as what happens, for example, on the ocean floor) a coupling occurs between evanescent SP-waves (*i.e.* a P-wave converted from an S-wave) in the soft layer and Rayleigh-type waves on the interface. The coupling condition is [22]

$$0.91\beta_1 < \alpha_0 = v, \tag{1}$$

where v is the phase velocity of the coupled mode. In the case of the Mexico City basin, this is satisfied since $\beta_1 \sim 100\,\mathrm{m/s}$ and $\alpha_0 = 1500\,\mathrm{m/s}$ is the sound velocity. The coupling occurs when the phase velocity of the dispersive Rayleigh waves is equal to the sound velocity α_0. This happens in Mexico City at a dominant frequency $\nu \approx 0.4\,\mathrm{Hz}$ which corresponds to a period $T = 2.5\,\mathrm{s}$. The coupled mode has many features in common with what is known as an Airy phase [21]. In particular, they are monochromatic and of long duration. We have called this mode a PR mode [22].

Once established, the PR mode reflects at the soft-clay boundaries due to the large impedance contrast between the clays and the sediments surrounding them, and the very fact that the boundary layer on and near which it lives, terminates at this boundary. These surface waves have to be evanescent outside the interface, they imply horizontal compression movement in the soft clay above the interface, and thus strongly couple to several normal modes in the ancient lake bed located in the horizontal plane. These modes provide the sea of complicated states [17,23,24].

One can understand easily the appearance of the PR-wave in the following steps:

1) When the SV-wave arrives to the interface between the soft clay and the sediments, a Rayleigh wave is created at the interface.

2) The Rayleigh wave travels along the interface and reflects at the end of the soft clay (in the sediment).

3) Since the Rayleigh wave is dispersive, it travels with a plethora of velocities given by the dispersion relation.

4) When eq. (1) is fulfilled, a Rayleigh wave exists that travels with the same velocity as the P-wave. Then the Rayleigh and the P-waves are coupled. The Rayleigh wave gives energy to the P-wave; this is the PR-wave.

5) The P-wave is evanescent in the upper direction and reaches to the free surface.

The normal-mode amplitudes ϕ_i and frequencies ν_i can be calculated solving the 2D Helmholtz equation in the horizontal plane

$$\nabla^2\phi_i + k_i^2\phi_i = 0 \tag{2}$$

in the region of the ancient Tenochtitlan lake (see figs. 1 and 5) with Neumann boundary conditions $\hat{n}\cdot\nabla\phi_i = 0$, where $\hat{n}$ is a vector normal to the boundary. The velocity of the P-wave is taken as α_0. The numerical results were obtained using the finite-element method with linear polynomials [25]. The region was discretized using 8272 points located in a rectangular grid. The size of the grid is 118 m. We verified the accuracy of the program to be better than one percent using a region with a rectangular boundary. Note that the Neumann conditions are somewhat arbitrary, but a similar calculation with Dirichlet conditions yields qualitatively the same results. Quantitative details cannot be expected, as we neither know the exact boundary conditions nor their exact location to a precision that would make the difference relevant.

In fig. 5 some wave functions with a normal mode frequency $\nu_i \approx 0.4\,\mathrm{Hz}$ are shown for the area containing the soft clays in Mexico City (see ref. [24]). The difference in frequency of these normal states is of the order 0.01 Hz. The states ϕ_i produce regions of constructive and destructive interference, a fact which explains the “pockets of damage” always observed in Mexico City earthquakes [24].

Since the source of the seismic waves is far away from the basin (over 200 kms), we represent the PR mode

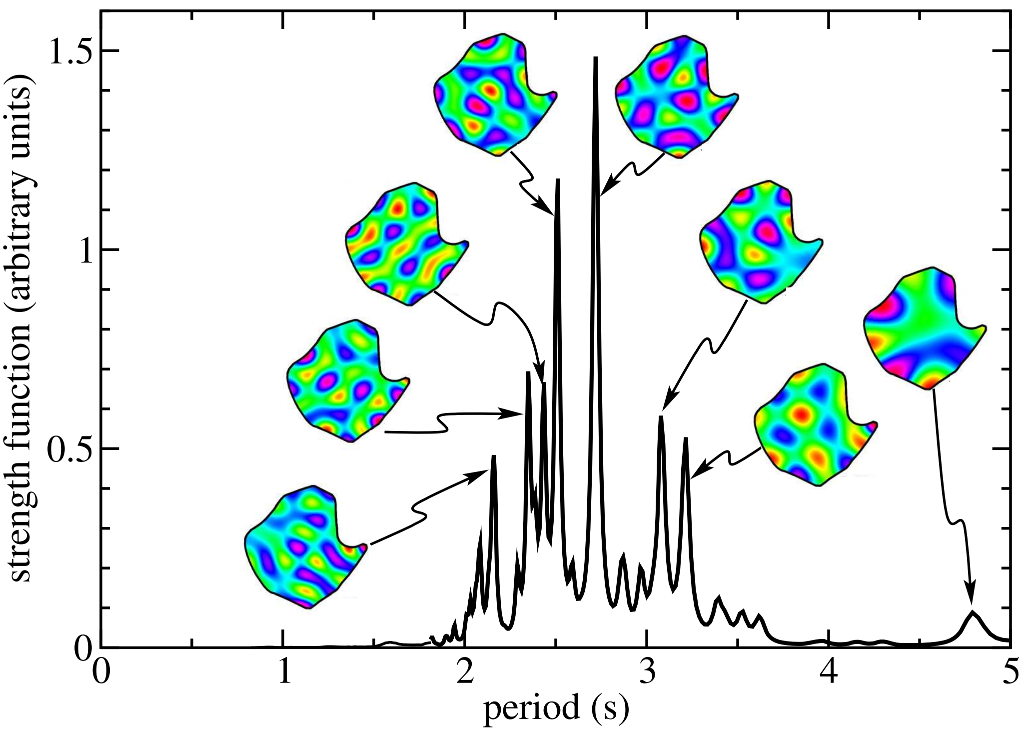

Fig. 5: (Color online) Strength function A as a function of the period $T_i = 1/\nu_i$. Some wave amplitudes are shown for the soft-clay area corresponding to the Tenochtitlan lake of fig. 1. They correspond to different peaks in the strength function.

by a plane wave $\exp(i\mathbf{k}\cdot\mathbf{r})$, where the direction of $\mathbf{k}$ varies for different earthquakes corresponding to different epicenters. Here $\mathbf{r}$ fixes a point within the bounded soft terrain in the horizontal XY-plane. The magnitude of $\mathbf{k}$ for the PR mode is $k = 2\pi\nu/\alpha_0$. The spreading of this mode among the normal modes ϕ_i is then given by

$$A(\nu_i) = \left| \int \exp(i\mathbf{k}\cdot\mathbf{r})\phi_i(x,y)\mathrm{d}x\mathrm{d}y \right|^2. \qquad (3)$$

In fig. 5 we plot the resulting spreading width $A(T_i)$ and compare it in fig. 3 with the Fourier spectrum magnitude observed in the earthquake of 1997. The angle of incidence is estimated from the direction of a line joining the lake basin with the epicenter. For this calculation we use a typical quality factor $Q = 80$. As can be seen comparing this figure with the experimental doorway state of fig. 3, the agreement is very good. Fourier spectra corresponding to older earthquakes measured with poor resolution may be obtained taking a smaller value of the quality factor Q. The change in the quality factor allows us to predict doorway states with increasing resolution for future improvements in the accelerometric grids. The result for several values of Q is shown in fig. 6.

In fig. 7 we show the spreading width $A(T_i)$ as a function of the angle of incidence. The continuous line corresponds to the earthquake of 1997, January 11. As can be seen in this figure, a shift in the direction of incidence affects the strength function slightly by changing the amplitude of each peak; in all cases, however, the strength function phenomenon is clearly present.

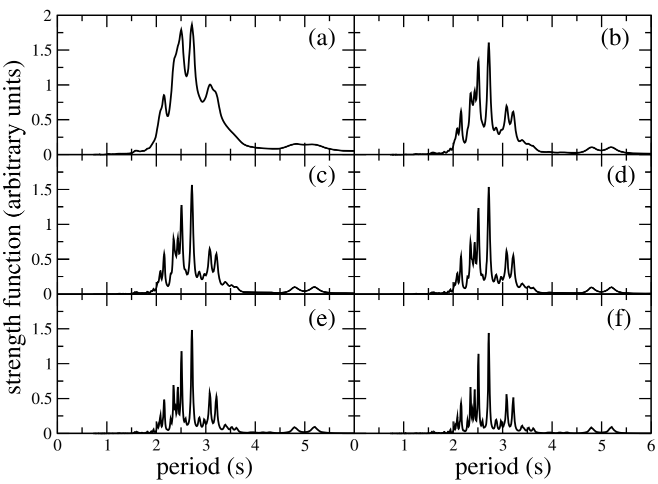

Fig. 6: The strength function for different values of the quality factor: (a) $Q = 20$; (b) $Q = 40$; (c) $Q = 50$; (d) $Q = 60$; (e) $Q = 80$ and (f) $Q = 100$. Giant resonances are particularly evident for low values of Q.

Conclusions. – We have shown that the Rayleigh wave at the interface between soft grounds and harder sediments acts as a typical doorway state and thus permits the excitation of horizontal P-wave resonances in sedimentary basins with a harder sediment and a waterlogged surface layer. The doorway state mechanism explains the coupling of such modes to the incoming S-waves, which are the main bearers of energy after a seismic event. One may say that the doorway state is the culprit of the strong ground motion and the enormous damage observed during

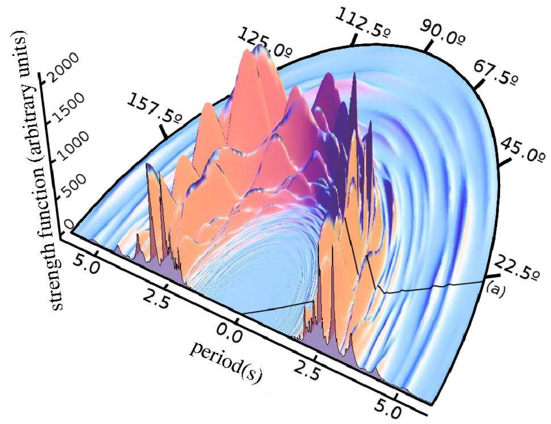

Fig. 7: (Color online) The strength function depends slightly on the incident angle (see fig. 1) from the epicenter of the seism. The line marked by (a) corresponds to the 1997 earthquake.

prolongued and large seismic events in urban sites built above mud grounds reclaimed from beaches and old lake beds.

The occurrence of doorway states in large classical systems is thus not only demonstrated in a practical example on a scale four to five orders of magnitude larger than displayed in the micro-wave experiments, but it is also shown to be of fundamental importance to understand and thus prevent seismic damage in the future. An option is to create a norm in the construction regulations to avoid buildings in the soft clays with resonant periods between 2 s and 4 s. This in turn emphasizes the universal importance of the doorway state mechanism, which now is established on scales reaching from fermis to tens of kilometers. Simultaneously, we see that the doorway states involved may result from very different physical conditions, that reach far beyond the symmetry considerations originally contemplated.

* * *

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