Energy transfer of a chaotic particle in a classical oscillating potential barrier

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Abstract

We study the classical dynamics of a particle inside a rigid box with an internal, arbitrarily located, square-potential barrier that oscillates periodically in time. We derive an area-preserving map that describes the dynamics of the particle, and show that it presents periodic, quasiperiodic and chaotic behavior, and generally mixed phase-space structure. We analyze the properties of the particle's classical "dwell time" to traverse the oscillating potential barrier. We also discuss in some detail the orbits and the time dependence of the energy transfer. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

The problem of dissipation, friction and energy transfer between classical or quantum systems has traditionally been studied phenomenologically. Recently, a more basic approach to these problems has appeared based on classical or quantum chaotic ideas [1,2]. In this paper we consider the energy transfer between a particle and an oscillating potential in a classical one-dimensional model that exhibits some of the basic physical features of this problem.

2. The model and the map

We consider the classical dynamics of a particle in a one-dimensional box, with an oscillating rectangular potential barrier, centered at different positions inside the box. The Hamiltonian of this system is time dependent, thus total energy of the particle is
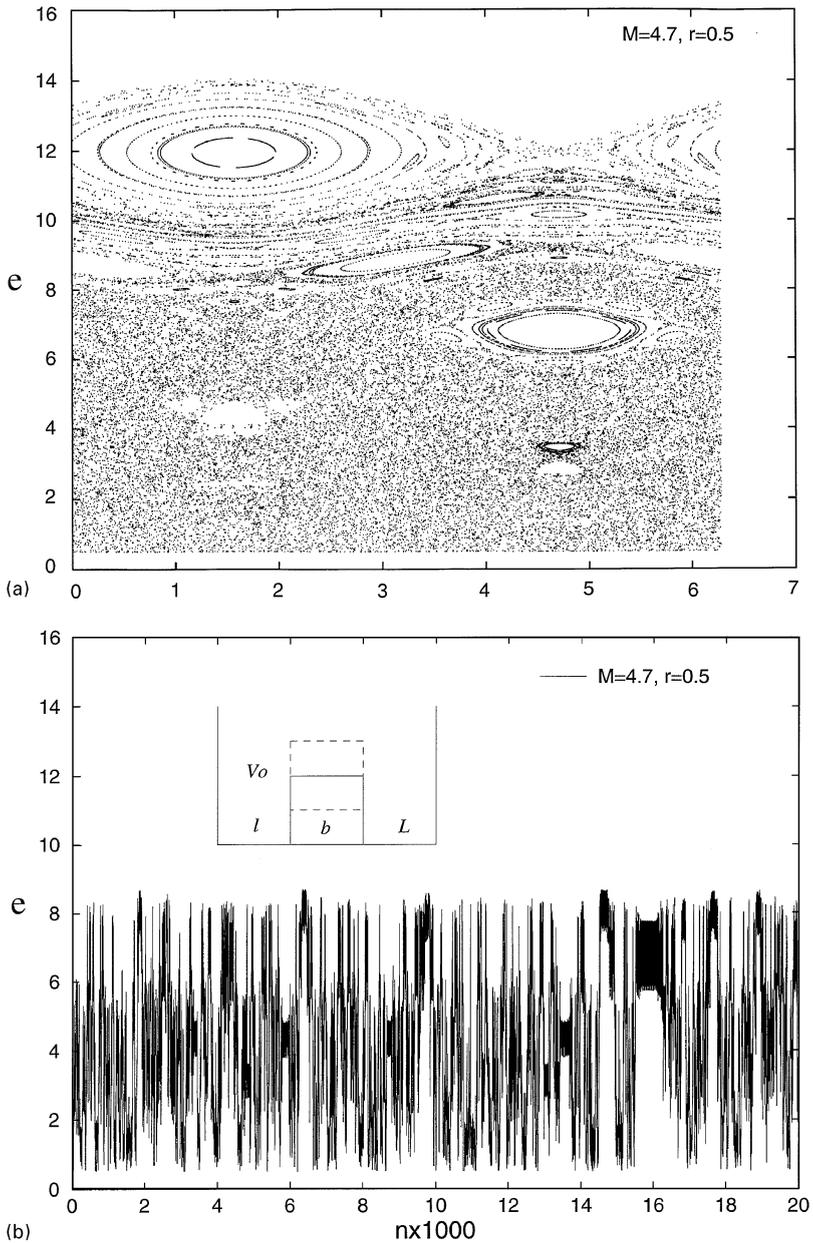
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Fig. 1. Here we show the energy vs. (a) phase and (b) discrete time, for $M = 4.7$ and $r = 0.5$.

not conserved. The Hamiltonian is given by $H(x, p, t) = p^2/2m + V_0(x) + V_1(x)f(t)$, where $f(t+\tau) = f(t)$ with τ the period of oscillation of the barrier. The potential $V_0(x)$ is infinite when $x < 0$ or $x > l + b + L$, and is constant (V_0) when $l \leqslant x \leqslant l + b$, otherwise is zero (see inset in Fig. 1b). The potential $V_1(x)$ is different from zero only inside

the interval $l \leqslant x \leqslant l + b$, where it is constant ($V_1$). The potential barrier inside the box then oscillates between $V_0 - V_1$ and $V_0 + V_1$, with period τ. When $V_1 = 0$, the motion is regular and the energy is conserved. However, when $V_1 f(t) \neq 0$, the dynamics becomes more complicated, being periodic, quasiperiodic and chaotic depending on parameter values.

To analyze the dynamics of H, as usual, it is convenient to derive a discrete map from the time t_n, when the particle hits the wall at $x = 0$, till the next time t_{n+1}, when it hits this wall again. Let us denote by E_n the energy of the particle immediately after the nth kick with the fixed wall at $x = 0$. We rescale time by the $f(t)$ period τ, and define the dimensionless quantities, $\phi_n = (2\pi/\tau)t_n$ and $e_n = E_n/V_0$. With these definitions we can write the corresponding map for the model:

$$e_{n+1} = e_n''' + 1 + rf(\phi_n + \Phi_n), \tag{1}$$

$$\phi_{n+1} = \phi_n + \Phi_n + \frac{2\pi M}{e_{n+1}^{1/2}} \pmod{2\pi}, \tag{2}$$

where $M = l/(w\tau)$, $r = V_1/V_0$ and $w = (2V_0/m)^{1/2}$. Here,

$$\Phi_n = 2\pi M \left(\frac{1}{e_n^{1/2}} + \frac{b}{l} \frac{1}{e_n'^{1/2}} + \frac{2L}{l} \frac{1}{e_n''^{1/2}} + \frac{b}{l} \frac{1}{e_n'''^{1/2}} \right), \tag{3}$$

$$e_n''' = e_n'' - 1 - rf \left(\phi_n + 2\pi M \left(\frac{1}{e_n^{1/2}} + \frac{b}{l} \frac{1}{e_n'^{1/2}} + \frac{2L}{l} \frac{1}{e_n''^{1/2}} \right) \right), \tag{4}$$

$$e_n'' = e_n' + 1 + rf \left(\phi_n + 2\pi M \left(\frac{1}{e_n^{1/2}} + \frac{b}{l} \frac{1}{e_n'^{1/2}} \right) \right), \tag{5}$$

$$e_n' = e_n - 1 - rf \left(\phi_n + 2\pi M \frac{1}{e_n^{1/2}} \right). \tag{6}$$

This map is area-preserving [3] since one can show that it has Jacobian equal to one.

3. Numerical results

We have numerically iterated the map discussed above under different parameter conditions, using $f(\phi) = \sin(\phi)$. There are four relevant dimensionless parameters: The width of the barrier b/l scaled with the length l; the ratio of the lengths L/l; the ratio of the amplitude barrier oscillation scaled to its height $r = V_1/V_0$; and $M = l/(w\tau)$. The M parameter is the ratio of the time of flight at the left of the barrier, with velocity w, and the period of oscillation of the barrier. That is, M measures the number of oscillations of the barrier from the time the particle leaves the wall at $x = 0$ till it arrives at the left side of the barrier. In what follows, we choose the following values: $b/l = 0.2$ and $L/l = 1.0$, which correspond to the oscillating barrier at the center of the box.

In Fig. 1a we show a typical phase space plot (e_n, ϕ_n) for $M = 4.7$ and $r = 0.5$. We plot several solutions that correspond to different types of orbits. We can clearly see
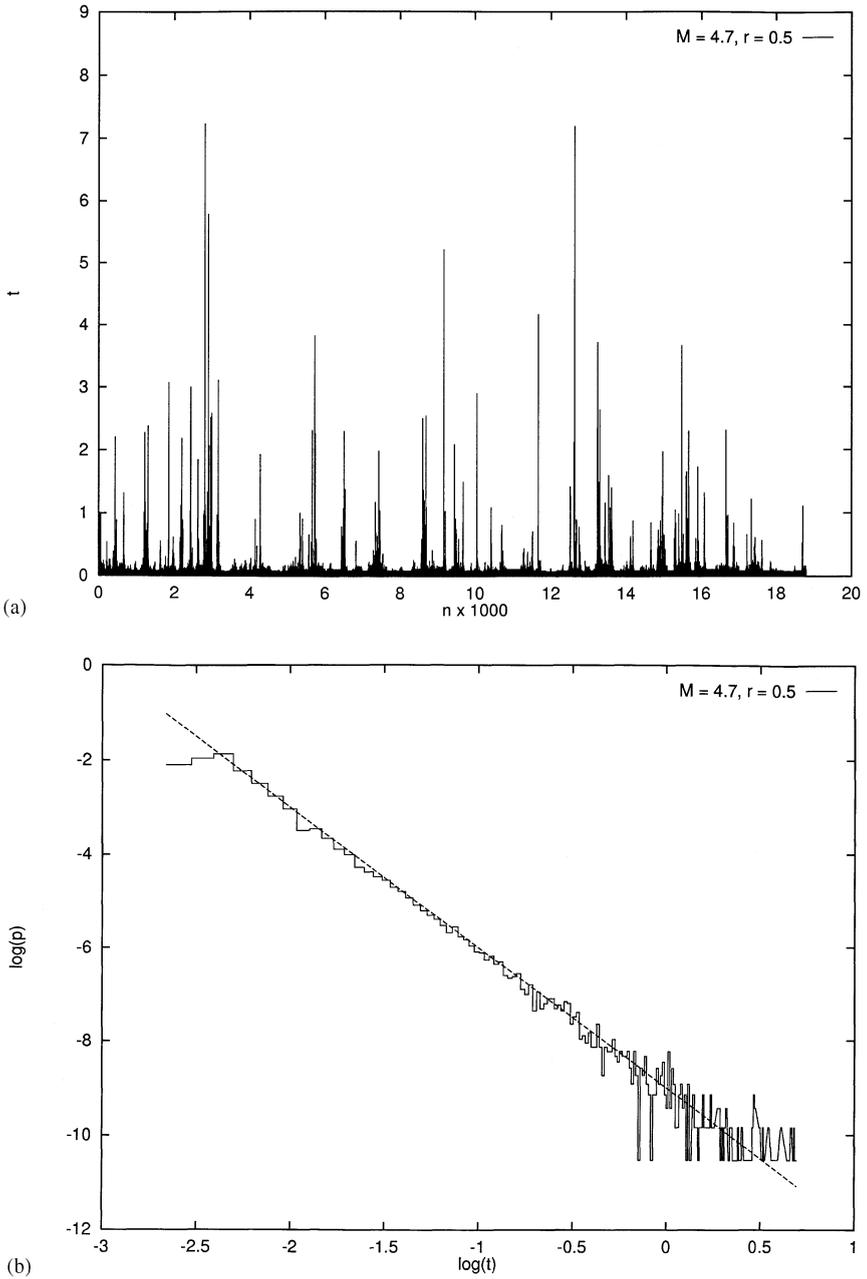
(a)

(b)

Fig. 2. (a) The dwell time as a function of the number of crossings and (b) log–log plot of the dwell-time distribution. The parameter values are the same as in Fig. 1a.

in this case that we have a mixed structure phase space, with periodic, quasiperiodic and chaotic orbits. In Fig. 1b we show the energy, corresponding to a chaotic orbit in Fig. 1a, as a function of n. We see that e_n varies erratically with n, but is bounded due to the presence of KAM barriers seen in Fig. 1a. As we increase M and/or r, we can gain more energy as it has access to higher energy regions in phase space.

In Fig. 2a we show the dwell time vs the number of crossings. The dwell time is defined as the time the particle takes to cross the oscillating barrier region, and is given in dimensionless units by $t_D = b/lu$. In Fig. 2b we present a log–log plot of the distribution of dwell times. From this figure, we see that we can fit the distribution to the power-law form $p(t) \sim t^{-3}$. The straight line has a slope very close to -3. This exponent appears to be the same for a broad range of parameter values. This power-law distribution for the limited range of times considered may be an indication of a scale invariance, and only seems to occur for chaotic orbits.

4. Conclusions

In this paper, we presented preliminary results of a study of the dynamics of a particle inside a rigid box with an internal oscillatory potential barrier. This can be taken as a model for different interesting physical situations. The study was based on the analysis of an area-preserving map that describes the particle dynamics. We found that the particle can gain or loose energy in regular or in erratic or chaotic ways, but the energy gain is always bounded by KAM barriers. On the other hand, the dwell time appears to exhibit a scale invariant-like structure that was discussed in terms of a power-law distribution of dwell times.

References

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