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Current reversals in chaotic ratchets: the battle of the attractors

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Abstract

We present the effect of current reversals in inertial chaotic ratchets, without changing a control parameter. This novel situation can occur when we have multiple coexisting attractors in phase space that transport particles in opposite directions. We analyze this effect for the case of two coexisting attractors: a chaotic attractor that generates a positive current and a periodic attractor that generates a negative current.

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1. Introduction

Recent advances in non-equilibrium statistical physics have revealed various instances of the surprising phenomenon of noise enhanced order, such as stochastic resonance [1,2], Brownian motors or noise-induced transport [3–7]. These remarkable phenomena occur due to the constructive role of noise in nonlinear dynamical systems [8].

Noise-induced, directed transport in a spatially periodic system in thermal equilibrium is ruled out by the second law of thermodynamics. Therefore, in order to generate transport, the system has to be driven away from thermal equilibrium by an additional deterministic or stochastic perturbation. In the most interesting situation, these perturbations are unbiased, that is, the temporal, spatial or ensemble averages of the associated forces are required to vanish. Besides the breaking of thermal equilibrium,

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another important requirement to get directed transport in a spatially periodic system is the breaking of the spatial inversion symmetry. We speak then of Brownian motors, ratchet devices, or, in the biological realm, of molecular motors. These so-called ratchets can be modeled, for instance, by considering the nonlinear dynamics of a particle in a spatially periodic asymmetric potential and acted upon by an external time-dependent force of zero average.

The recent burst of interest in this area is motivated by a series of scientific and technological reasons in devising and understanding noise-induced separation and pumping techniques. The activity has been so intense that in less than a decade several hundred papers in this area have been written. See the review article by Reimann [9] for a thorough review of the field.

Although the vast majority of the literature in this field considers the presence of noise, very recently there has been some interest in understanding in detail the transport properties of classical *deterministic inertial ratchets* [10–36]. These ratchets have in general a classical chaotic dynamics that modifies the transport properties. The implications of this chaotic dynamics in deterministic ratchets have recently been addressed in the quantum domain, together with the possible connection with quantum chaos [37–42]. In particular, in a recent paper [11], we established a connection between the current generated in a deterministic rocking ratchet and the bifurcation diagram associated with the chaotic dynamics. When we plot the current, defined as an average velocity, against a control parameter of the system, we found multiple current reversals; on the other hand, we obtained the bifurcation diagram for the velocity as a function of the same control parameter. We noticed a clear connection between these two plots and by doing a close comparison we found that the current reversals are associated with bifurcations. In many cases, these reversals occur in crisis bifurcations from a chaotic to a periodic regime.

It is worth mentioning that this prediction has been verified qualitatively in a recent experiment done by Carapella et al. [36], using the ratchet effect for a relativistic flux quantum trapped in an annular Josephson junction embedded in an inhomogeneous magnetic field.

The phenomenon of current reversals in ratchets is well documented so far in a wide variety of systems; however, in most of these cases it is necessary to vary a control parameter in order to obtain the reversals. Here we analyze an instance in which it is possible to reverse the direction of the current in a ratchet, *without changing any parameter*.

In the parameter range studied in Ref. [11], we found a single chaotic attractor in phase space, and the current reversal occurs only when we vary a control parameter. However, a different situation takes place when one finds multiple attractors coexisting in phase space. Each of these attractors (periodic or chaotic) have their own basin of attraction and, in general, one attractor can transport particles in some direction while another attractor transport particles in the opposite direction. It is possible, then, to obtain current reversals without varying any parameter. We can reverse the current simply by changing slightly the initial conditions in phase space and, in this way, changing from one basin of attraction to another. It is important to mention that Flach and coworkers [16,19] have analyzed the case of dissipation close to the Hamiltonian limit (small damping) where one can find coexisting limit cycle attractors. They obtained finite currents due to the symmetry breaking of the corresponding basins of attraction.

2. The ratchet model

Here we analyze a regime of parameters where there are two coexisting attractors in phase space: a chaotic and a periodic attractor (limit cycle). The chaotic attractor generates a current in the positive direction and the periodic attractor generates a negative current. Each attractor has its own basin of attraction in phase space and the boundary of these basins has a fractal structure [43].

Let us consider the same equation of motion studied in Refs. [11–13], that is, a one-dimensional problem of a particle driven by a periodic time-dependent external force, under the influence of an asymmetric periodic potential of the ratchet type. The time average of the external deterministic force is zero. The dimensionless equation of motion is given by

$$\ddot{x} + b\dot{x} + \frac{\mathrm{d}V(x)}{\mathrm{d}x} = a\cos(wt)\,, \tag{1}$$

where b is the dimensionless friction coefficient, $V(x)$ is the external asymmetric periodic potential, a is the amplitude of the external force and w is the ratio of the frequencies of the external driving force and the linear motion around the minima of the potential. The ratchet potential is given by

$$V(x) = C - \frac{1}{4\pi^2\delta}\left(\sin 2\pi(x - x_0) + \frac{1}{4}\sin 4\pi(x - x_0)\right)\,, \tag{2}$$

where the potential is shifted by an amount x_0 in order that the minima of the potential are located at the integers, and δ is defined by $\delta = \sin(2\pi|x'_0|) + \sin(4\pi|x'_0|)$.

This potential is depicted in Fig. 1. As in Ref. [11], the constant C is such that $V(0) = 0$, and is given by $C = -(\sin 2\pi x_0 + 0.25\sin 4\pi x_0)/4\pi^2\delta$. There are many x_0 that can be chosen. In this case we take, $x_0 \simeq -0.19$, and therefore $\delta \simeq 1.6$ and $C \simeq 0.0173$.

The extended phase space in which the dynamics is taking place is three-dimensional, since we are dealing with an inhomogeneous differential equation with an explicit time dependence. This equation can be written as a three-dimensional dynamical system, that we solve numerically, using the fourth-order Runge–Kutta algorithm. The equation of motion Eq. (1) is nonlinear and thus allows the possibility of periodic and chaotic orbits.

Let us define the current as a time average of the velocity v over q periods T of the external force, where q is an integer number, and $T = 2\pi/w$. That is,

$$\bar{v} = \frac{1}{qT}\int_t^{t+qT} v(t')\,\mathrm{d}t'\,. \tag{3}$$

Since $v = \dot{x}$, we obtain $\bar{v} = [x(t + qT) - x(t)]/qT$. If the particle moves a distance p in a time qT, where p is another integer number, then we have the condition $x(t + qT) - x(t) = p$. Thus, we obtain that the current is given by $T\bar{v} = p/q$.

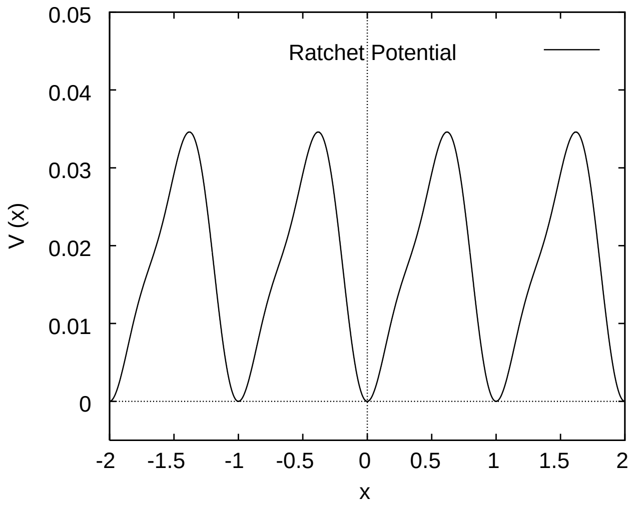

Fig. 1. The dimensionless ratchet periodic potential $V(x)$.

This means that we have *frequency locking* in which the current times T is a rational number p/q. We will see that this frequency locking takes place in a whole range of values of the parameter a.

3. Numerical results

We compare now the bifurcation diagram for the dynamical system given in Eq. (1) and the current, as a function of the amplitude a of the external force. We will fix the other two parameters as $b = 0.1$ and $w = 0.67$ [11]. In Fig. 2a we plot the bifurcation diagram in a range of the parameter a where there are two coexisting attractors: a chaotic and a periodic one. We notice that there is a crisis bifurcation around $a = 0.1563$ which destroys the chaotic attractor, while the periodic attractor is present throughout the whole range, having a negative velocity $v \simeq -0.125$. In Fig. 2b we show the current in the same range of a. We are plotting the rescaled current $T\bar{v}$, which is equal to the ratio p/q due to frequency locking. We observe in this figure a 1:1 resonance for the chaotic transport and a 2:1 resonance for the periodic one. The chaotic transport, with $T\bar{v} = 1$, is obtained using the initial conditions $x_0 = -0.10$, $v_0 = 0.25$, and the periodic transport, with $T\bar{v} = -2$, using $x_0 = 0.43$, $v_0 = -0.12$. These initial conditions are located near the attractors in their corresponding basins of attraction.

In Fig. 3 we show the phase space for $a = 0.156$, where we see the coexisting chaotic and periodic attractors. This attractors were calculated confining the dynamics *modulo 1*, using the periodicity of the potential $V(x + 1) = V(x)$. That is, even though the orbits are transporting particles from one well to the next, we collapse the motion to a unit cell in x between -0.5 and 0.5.

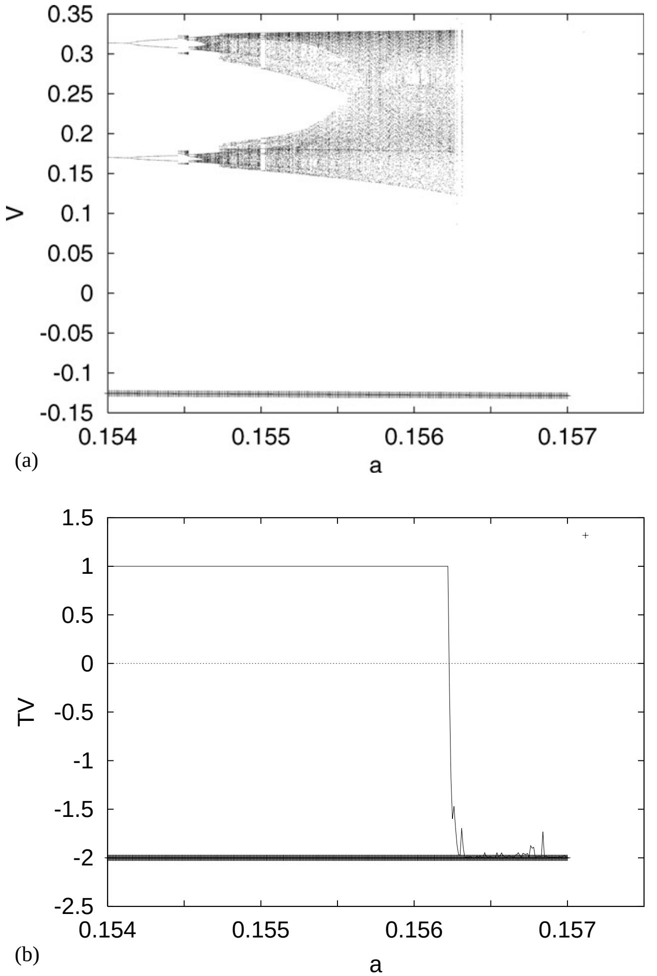

Fig. 2. For $b = 0.1$ and $w = 0.67$ we show: (a) The bifurcation diagram as a function of a, and (b) The rescaled current $T\bar{v}$ as a function of a.

In Fig. 4 we show the trajectories associated with each attractor. We see that the chaotic attractor generates a positive current and that the periodic attractor generates a negative one. This result implies that we can reverse the direction of the current by choosing the appropriate attractor.

Since we have these two attractors coexisting in phase space, we must have two basins of attraction. In general, one expects that these basins are intertwined in a complex way, and that the basin boundary is a fractal set [43]. By choosing the initial conditions we can select the desired basin and the corresponding attractor. In Fig. 5 we show the basins of attraction for the two attractors. The black region corresponds to the basin of the periodic attractor (negative current) and the white region corresponds

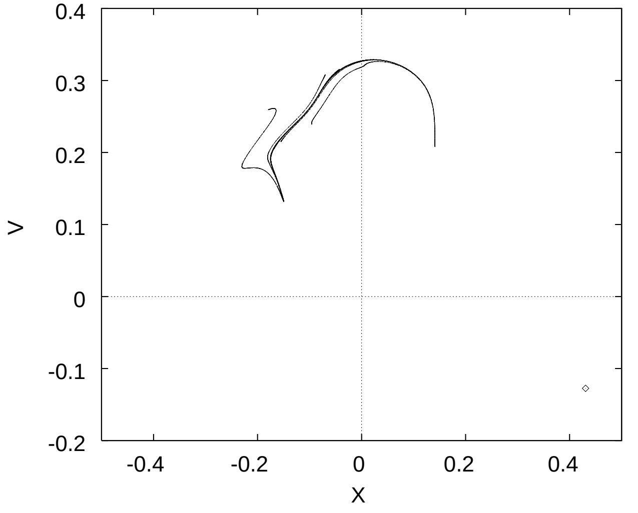

Fig. 3. For $b = 0.1$, $w = 0.67$ and $a = 0.156$ we show two coexisting attractors: a chaotic attractor (positive current) and a periodic attractor (negative current).

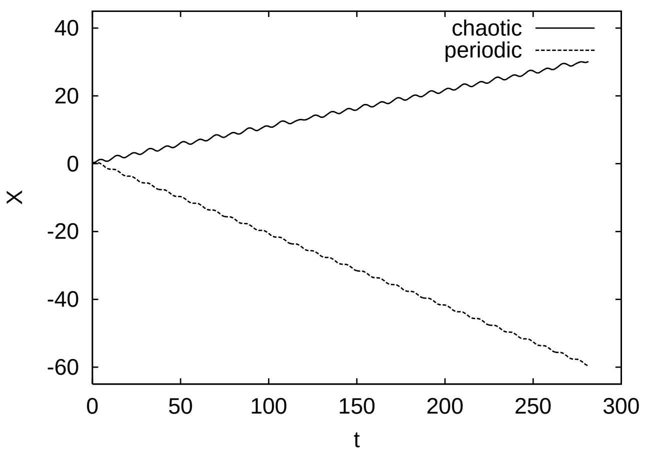

Fig. 4. For $b = 0.1$, $w = 0.67$ and $a = 0.156$ we show two trajectories: Positive current (full line) generated by the chaotic attractor; negative current (dashed line) generated by the periodic attractor.

to the basin of the chaotic attractor (positive current). For this figure we take $a = 0.156$ and we have plotted more than 10^6 initial conditions. It is apparent from Fig. 5 that the intertwined character of the basin boundary implies an extreme sensitivity to initial conditions. That is, even though we try to select a desired attractor, a minor error in the initial condition can result in the selection of the other one. Of course, we can use this property to separate particles using these attractors. For instance, we can place an

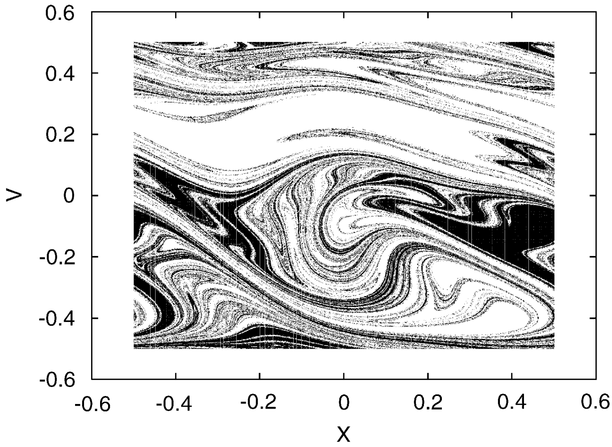

Fig. 5. For $b = 0.1$, $w = 0.67$ and $a = 0.156$ we show the basins of attraction of the two coexisting attractors. The white region corresponds to the chaotic attractor (positive current) and the black region corresponds to the periodic attractor (negative current).

ensemble of particles, with zero velocity, around the origin; those particles located in the white basin will move to the right, and those in the black basin will move to the left.

As we increase the control parameter a, the chaotic attractor moves in phase space until it collides with the basin boundary and is destroyed by a boundary crisis bifurcation at $a_c \simeq 0.1563$ (see Fig. 2a). After this point, only the periodic attractor survives and, being the only attractor, it attracts all the points in phase space. This means that, independently of the initial conditions, the current becomes negative. However, this negative current is achieved only after a chaotic transient that transport particles to the positive direction in a chaotic fashion. The duration of this chaotic transient can be very large near the bifurcation and decreases when we move away from the critical parameter a_c [43]. This is the reason why we notice in Fig. 2b some fluctuations in the velocity after a_c. The transient time in this case is larger than 10^6 in units of T. We choose to keep this fluctuations in Fig. 2b to illustrate this issue.

4. Concluding remarks

In summary, we have presented a mechanism by which we can reverse the current in chaotic deterministic ratchets without changing a control parameter. This novel type of reversal exploits the fact that for some parameters there are multiple coexisting attractors in phase space. One can find situations where one attractor transport particles in one direction, whereas another attractor transport particles in the opposite direction. By choosing the appropriate initial condition one can select a particular attractor and in this way select the direction of the current. An important implication of this work is the following: so far, inertial ratchets has been used for separation of particles having different masses, lengths, friction coefficients, etc. These physical properties serve as control parameters to be varied in order to achieve the current reversal. However, the new scheme presented here can be used to separate even *identical* particles, since we only need to vary the initial conditions.

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