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Anomalous mobility and current reversals in inertial deterministic ratchets

Fernando R. Alatriste, José L. Mateos*

Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 México, D.F., México

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Abstract

We analyze the transport properties of inertial deterministic rocking ratchets in the presence of an external constant force. For small values of this load, we can obtain a positive current for a negative load, and vice versa. This phenomenon, in which the direction of the current is opposed to the sign of the external force, is a signature of anomalous negative mobility. We show that this anomalous mobility is possible in the deterministic case, and explain this phenomenon as current reversals associated to bifurcations in an inertial deterministic rocking ratchet in the presence of an external load.

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1. Introduction

A ratchet is a system in which an asymmetry is built in to rectify a signal of zero average [1–4]. Among the many kinds of ratchets studied recently, an important class refers to classical deterministic inertial ratchets in which the dynamics does not have any randomness or stochastic elements [5–7]. Since a ratchet is a non linear system, in some cases its deterministic dynamics can exhibit chaotic motion. This is indeed the case when we consider inertial effects, for instance, in a one-dimensional rocking ratchet [8–11]. This kind of ratchet can be modeled by a particle with inertia and friction on a one-dimensional asymmetric ratchet potential and acted by a harmonic time-dependent force of zero average. For this case, a surprising phenomenon appears: current reversals [8–20]. These reversals of the average velocity where explained in Ref. [9], by establishing a connection of the current with the bifurcation diagram as a function of a control parameter. It has been found that even in the case of bifurcations from periodic to periodic orbits it is possible to found current reversals, or that in some other cases a tangent bifurcation is not associated with a current reversal. However, we can say that, in general, there is a strong connection between the current in a deterministic ratchet and its bifurcation diagram. Usually, the different types of bifurcations are linked with sudden changes in the current: either

*Corresponding author. Tel.: +52 55 5622 5130; fax: +5 55 5622 5015.
E-mail address: mateos@fisica.unam.mx (J.L. Mateos).

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current reversals, abrupt changes from a finite current (positive or negative) to a zero current, or just sudden changes in the absolute value of the current.

In Ref. [9], a deterministic rocking inertial ratchet was studied and it was established that current reversals can be associated with tangent bifurcations from chaotic to periodic orbits, that lead to intermittency and anomalous diffusion. In this case, the control parameter of the bifurcation diagram was the amplitude of the harmonic forcing. As in many other nonlinear dynamical systems, the dynamics of a deterministic ratchet can be even more complex. For other values of the amplitude of forcing, we notice in the bifurcation diagram signatures that indicate the possibility of coexisting attractors in phase space. In Ref. [11], the effect of current reversals in inertial chaotic ratchets, without changing a control parameter, was studied. This situation, which was termed the battle of the attractors, can occur when we have multiple coexisting attractors that transport particles in opposite directions. Each attractor has its own basin of attraction that is selected through initial conditions.

The study of inertial deterministic ratchets is nowadays an important subfield in its own, due to recent experiments on vortex and SQUID ratchets, where the importance of inertial deterministic ratchets has been stressed [21–29]:

We will be dealing with a deterministic inertial ratchet, but with an additional external constant force that provides a fixed tilt in the ratchet potential, and thus the dynamics can be represented by an inertial particle in a washboard potential. In this paper, we will show that a deterministic particle is able to exhibit anomalous mobility in a one-dimensional periodic ratchet potential. That is, the current has the opposite sign of the external constant force, for small values of the latter. Previous studies of anomalous and absolute negative mobility considered nonequilibrium systems with stochastic forces or thermal noise, in the case of interacting Brownian particles [30–34] and single particle stochastic models [35–45]. More recently, the case of absolute negative mobility in a one-dimensional periodic and symmetric potential, including the deterministic case, was studied [46,47]. Therefore, this one-dimensional deterministic model can be considered the simplest case exhibiting anomalous mobility in a ratchet system.

2. Anomalous mobility in inertial tilted ratchets

Let us consider the one-dimensional problem of a particle driven by a periodic time-dependent external force in an asymmetric periodic ratchet potential. Here, we do not take into account any sort of noise, meaning that the dynamics is deterministic. Two additional forces act on the particle: a dissipative force proportional to the velocity, and an external constant force. We thus deal with a rocked deterministic tilted ratchet in the underdamped case that obeys the dimensionless equation of motion:

$$\ddot{x}+\dot{x}+\frac{\mathrm{d}V(x)}{\mathrm{d}x}=F+F_D\cos(\omega_D t), \tag{1}$$

where $V(x)$ is the asymmetric periodic ratchet potential, F is a constant force, F_D and ω_D represent the amplitude and the frequency of the external driving force, respectively. The dimensionless ratchet potential is given by

$$V(x)=V_0\left[C-\sin 2\pi(x-x_0)-\tfrac{1}{4}\sin 4\pi(x-x_0)\right], \tag{2}$$

where V_0 is the amplitude, and C is an arbitrary constant. The potential is shifted by an amount x_0 in order that the minimum of the potential is located at the origin, and is depicted in the inset of Fig. 1. The constant C is such that $V(0)=0$, and is given by $C=-(\sin 2\pi x_0+0.25\sin 4\pi x_0)$. We choose, $x_0\simeq -0.19$, see [9].

When $F_D=0$, we have a tilted ratchet that obeys the equation of motion: $\ddot{x}+\dot{x}+\mathrm{d}V(x)/\mathrm{d}x=F$. The tilted (time-independent) washboard potential is, in this case, $U(x)=V(x)-Fx$, see Fig. 1.

3. Numerical results

In this section we will solve numerically the equation of motion for the rocking tilted inertial ratchet. We use the fourth-order Runge-Kutta algorithm to solve the differential equation (1). We will fix throughout the

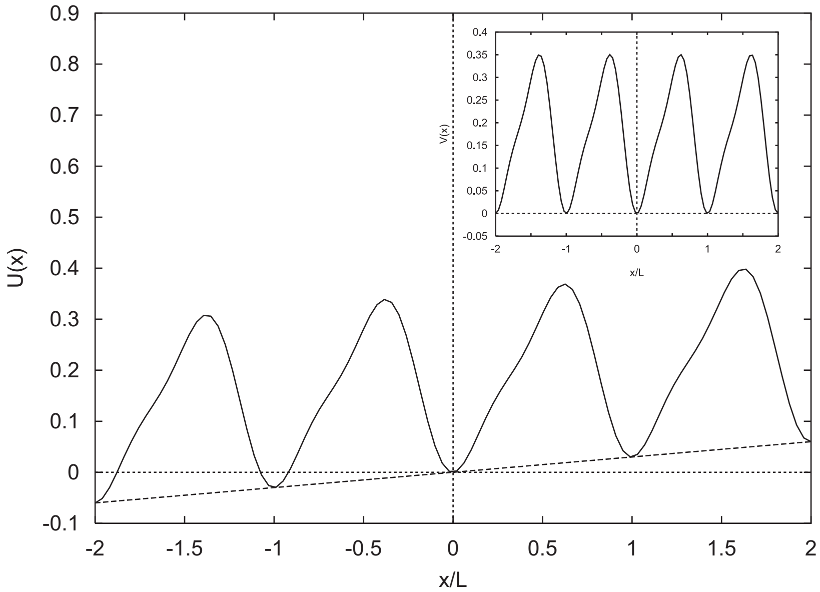

Fig. 1. The tilted washboard ratchet potential for small values of the tilt where the effect of anomalous mobility is observed in the deterministic dynamics.

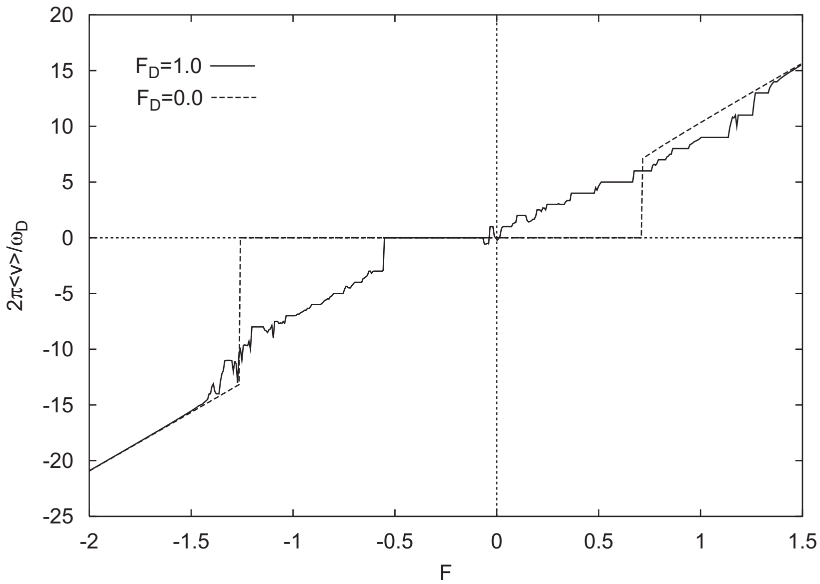

Fig. 2. The scaled average velocity $2\pi\langle v\rangle/\omega_D$ as a function of the amplitude of the external force F. The dashed line indicates the case when the driving harmonic force is absent ($F_D = 0$) and the continuous line is the case when this force is $F_D = 1.0$. In both cases we used $\omega_D = 0.6$.

paper the amplitude of the ratchet potential as $V_0 = 1/2\pi$. With this value, the critical tilt to the right is $F_c^R \simeq 0.71$ and to the left $F_c^L \simeq -1.26$.

In Fig. 2, we depict the time average velocity, scaled with the driving frequency, $2\pi\langle v\rangle/\omega_D$ as a function of the tilt F. The dashed line shows the case without periodic driving ($F_D = 0$) and corresponds to the fixed washboard potential $U(x) = V(x) - Fx$. Notice that the current is zero until we arrive at the critical tilt F_c^R to the right or to the critical tilt F_c^L to the left. This step of zero current is not centered around the origin, due to the asymmetry of the ratchet potential. For values greater than F_c^R we have a finite current that increases monotonically with F. Of course, for values less than F_c^L we obtain a negative average velocity that decreases for negative values of the tilt. When the periodic driving is present, the current acquires a series of clearly defined steps for values of the current given by the ratio p/q, where p and q are integer numbers. In many cases, $q = 1$ and the average current is an integer. Therefore, a rational value of $2\pi\langle v\rangle/\omega_D = p/q$ means that $\langle\omega\rangle = (p/q)\omega_D$ for a whole range of values of the tilt, since the average frequency $\langle\omega\rangle = 2\pi\langle v\rangle$. This well-known phenomenon of frequency locking and synchronization has been explored recently for deterministic overdamped ratchets [48], and coupled inertial ratchets with time delay [49].

In Fig. 3, we show in detail the behavior of the current for small values of the tilt F around the origin. We notice that the current is positive even though the tilt is negative, and vice versa; a clear signature of anomalous negative mobility. For $F \simeq -0.03$, we notice that the current is constant in a whole range of values of F, indicating frequency locking; in this case $2\pi\langle v\rangle/\omega_D = 1$. That is, for negative values of the tilt, the current is positive and large. On the other hand, in a narrow range around $F \simeq 0.005$, the current is negative with $2\pi\langle v\rangle/\omega_D = -2$. Therefore, this figure clearly shows the effect of negative mobility in the case of a deterministic inertial ratchet. This figure also shows a very rich structure of other narrower steps revealing other instances of anomalous negative mobility, and a subtle structure of current reversals. Additionally, we also observe positive mobility for other values of the tilt F. Outside the range $-0.07 < F < 0.03$, we recover the usual situation in which the mobility is zero or has the same sign as the external tilt. Thus, it is only in a limited range of small values of F around zero that we expect to find the effect of negative mobility. To elucidate the richness of information in this figure, in what follows we will make a comparison of this response with the associated bifurcation diagram.

In order to understand the origin of the anomalous negative mobility in this deterministic ratchet, we will calculate the bifurcation diagram using the tilt F as the control parameter. To calculate this diagram we solve numerically Eq. (1) and obtain the velocity as a function of time $\dot{x}(t)$ and then plot the asymptotic value of this velocity as a function of F. In Fig. 4, we show the bifurcation diagram as a function of F. Notice the rich structure of the dynamics, showing periodic and chaotic orbits, as is usually the case for inertial deterministic ratchets [9]. In particular, for a critical value around $F = -0.035$, we notice a tangent bifurcation from a chaotic to a period orbit, that is responsible for current reversal from negative to positive current. This current is positive and constant for a whole range between $-0.035 < F < -0.0175$ leading to the phenomenon of

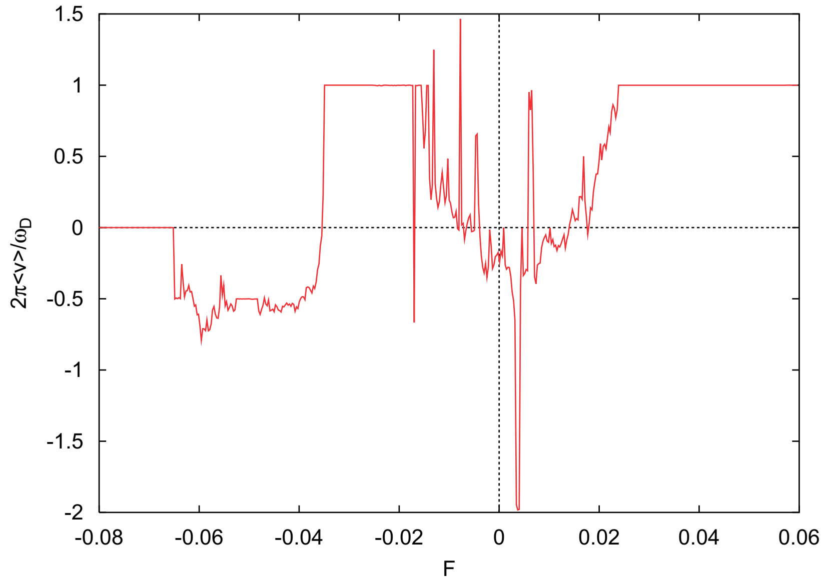

Fig. 3. The detailed structure of the scaled average velocity $2\pi\langle v\rangle/\omega_D$ as a function of the amplitude of the external force F, around $F = 0$, for $F_D = 1.0$ and $\omega_D = 0.6$. Notice the step at $2\pi\langle v\rangle/\omega_D = 1$ in the interval $-0.035 < F < -0.0175$, indicating anomalous negative mobility.

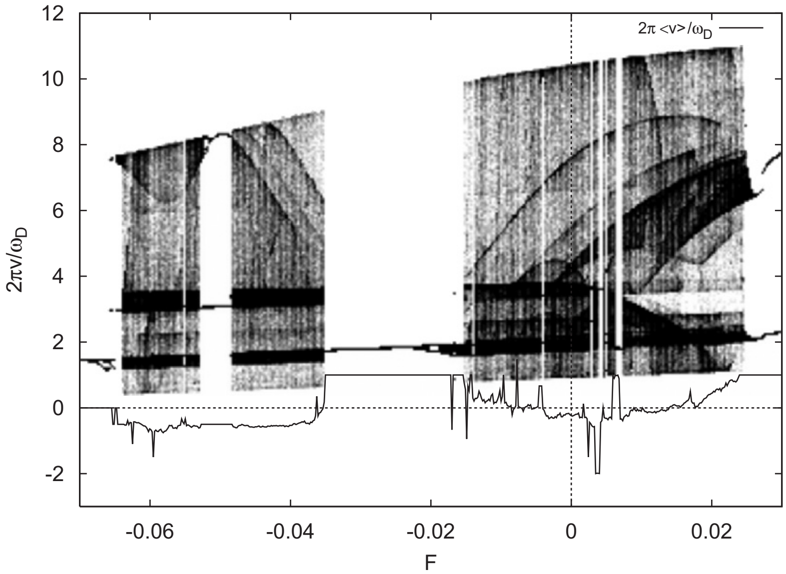

Fig. 4. Bifurcation diagram as a function of the amplitude of the tilt F, in the same range as Fig. 3, around $F = 0$, for $F_D = 1.0$ and $\omega_D = 0.6$.

anomalous negative mobility. Another instance of current reversals associated with bifurcations leading to negative mobility can be observed for positive tilt around $F \simeq 0.005$. Notice however that in some other cases, for instance around $F \simeq -0.05$, we have another periodic window in the borders of which we have bifurcations from chaotic to period orbits that does not lead to current reversals and anomalous mobility. Additionally, it might be possible that some other kind of bifurcations may induce this anomalous mobility.

Therefore, the negative mobility can be interpreted as a current reversal from the expected direction of motion, and this reversal is associated with a tangent bifurcation from a chaotic to a periodic orbit in the bifurcation diagram, as has been shown in Ref. [9].

It is natural to wonder about the generality of this result for this model. Therefore, we have explored other values in the parameter space and found that the phenomenon of anomalous negative mobility appears only for small values of the tilt F. We have encounter the same dynamics of current reversals leading to anomalous mobility through bifurcations, for driving amplitudes in the interval $1 < F_D < 2$, and driving frequencies in the interval $0.5 < \omega_D < 1.0$. This allows us to infer that this effect might be robust, although it would be desirable to explore also other values of the parameters and even other forms for the ratchet potential.

In Fig. 5, we illustrate the chaotic and periodic attractors around the bifurcation point $F = -0.035$. For the chaotic attractor, $F = -0.035123$, and for the periodic attractor, $F = -0.035122$. The inset shows a typical trajectory climbing the tilted ratchet potential (positive current), for the negative value of the tilt $F = -0.035122$, characteristic of anomalous negative mobility. In this trajectory we notice that the particle moves one well to the right during exactly one period of the external forcing. This corresponds to a frequency locking where $p/q = 1/1$. Due to this 1:1 resonance, the scaled average velocity $2\pi\langle v\rangle/\omega_D = 1$, in a whole range of values of the tilt $(-0.035 < F < -0.0175)$, as depicted in Fig. 3. We show in Fig. 5 a chaotic and a periodic attractor. The periodic attractor correspond to a period one orbit and is illustrated as a thick dot near the point $(0.2, 1.8)$ in phase space. This occurs since we are plotting a Poincaré section using the period of the external forcing as the stroboscopic time. The phase space represented here has the topology of a cylinder, meaning that the velocity is not bounded, but the coordinate x is bounded due to the periodicity of the ratchet potential; in this representation we used x modulo one. In an extended (or unfolded) representation, we would see an unbounded coordinate x that corresponds to an open trajectory that transport particles, as shown in the inset. The chaotic attractor is represented as a set of dots that span through phase space. Since this attractor corresponds to a value of the tilt F very close to the bifurcation point, the density of points in the attractor tends to increase in the vicinity of the periodic attractor. We have here a typical tangent bifurcation where the

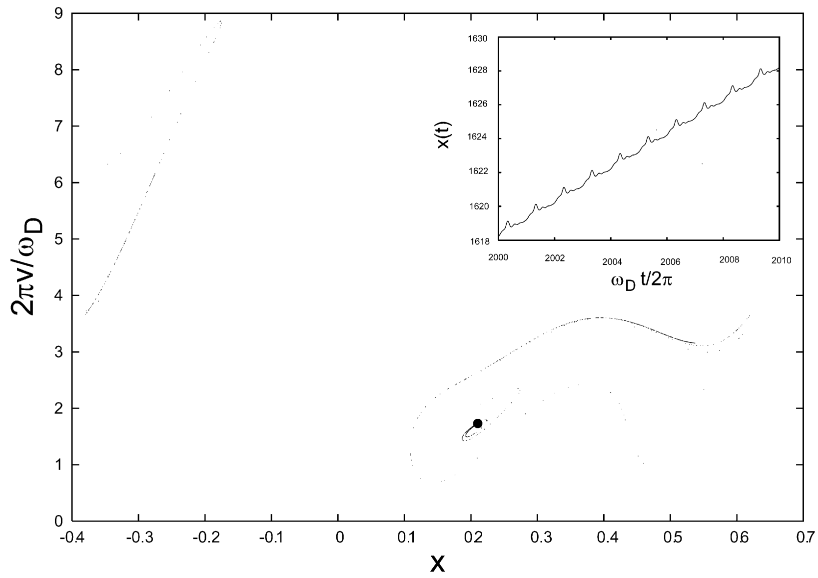

Fig. 5. Phase space illustrating the chaotic and periodic attractors around the bifurcation point $F = -0.035$, for $F_D = 1.0$ and $\omega_D = 0.6$. In the inset, we show a typical trajectory climbing the tilted ratchet potential; even though the tilt is negative, the current is positive.

chaotic attractor suddenly collapses to a single point (the periodic attractor) at the critical value of the tilt F. The dynamics corresponding to the chaotic attractor shown in Fig. 5 has an intermittent character, as discussed in detail in Ref. [9,10]. As mentioned before, at this bifurcation point, we have a current reversal that leads to a positive current for a negative tilt, and therefore to anomalous negative mobility.

4. Concluding remarks

In summary, we have analyzed the deterministic dynamics of a tilted inertial rocking ratchet as a function of the external constant force. We obtained the remarkable phenomenon of anomalous negative mobility in this deterministic case, that is, the direction of the current (average velocity) is opposed to the sign of the applied force. After analyzing the relationship between the current and the bifurcation diagram, taking the tilt as the control parameter, we established that this negative mobility can be interpreted as a current reversal from the expected direction of motion, associated with bifurcations between chaotic and period orbits. We found this connection in a whole range of values of the parameters of the model, such as the driving frequency of the periodic forcing and the driving amplitude. This fact allows us to infer that the phenomenon of anomalous negative mobility might be obtained not only for this particular model but also for other forms of the ratchet potentials.

Finally, it is worth mentioning that this effect is amenable to be explored experimentally using Josephson Junctions or vortex ratchets, where the equations of motion and the associated dynamics is similar to the nonlinear dynamics of inertial deterministic ratchets.

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